Università di Trieste – Facoltà d'Ingegneria.

Esercizi: foglio 33

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Esercizio 1 Si calcolino i seguenti integrali:

$$\text{a)}\quad \int_0^1 x^2 \cdot \operatorname{arcsen} x\, dx \qquad \text{b)}\quad \int_0^1 x \cdot 2^x\, dx;$$

$$\text{c)}\quad \int_2^4 \frac{\sqrt{x}-1}{\sqrt{x}+1}\, dx; \qquad \text{d)}\quad \int_0^1 \frac{e^{2x}-3e^x}{e^x+1}\, dx;$$

$$\text{e)}\quad \int_0^1 \sqrt{\frac{1+\operatorname{senh}^2(x)}{\operatorname{senh}(x)+2}}; \qquad \text{f)}\quad \int_0^{\frac{\pi}{4}} \frac{1+\operatorname{tg}^2(x)}{\cos^2(x)}\, dx.$$

Esercizio 2 Si calcoli una primitiva per ciascuna delle seguenti funzioni:

$$\text{a)}\quad \sinh(\alpha x)+\cosh(\beta x) \qquad \text{b)}\quad \log(1-x) \quad ; \qquad \text{c)}\quad \operatorname{sen}^2 x;$$

$$\text{d)}\quad \sqrt{4-x^2}; \qquad \text{e)}\quad \frac{\log x}{x}; \qquad \text{f)}\quad x\cdot \operatorname{sen}(x^2)\cdot e^{2x^2};$$

$$\text{g)}\quad x\cdot e^{\sqrt{x}}; \qquad \text{h)}\quad \operatorname{arcsen}(x); \qquad \text{i)}\quad \operatorname{arctg}(x) \quad .$$

Esercizio 3 Si calcoli una primitiva per ciascuna delle seguenti funzioni razionali:

$$\text{a)}\quad \frac{x}{3x-1} \qquad \text{b)}\quad \frac{x+3}{x^2+1}; \qquad \text{c)}\quad \frac{1}{x^4-4x^3};$$

$$\text{d)}\quad \frac{9x^4-6x^3+x^2+1}{1-6x+9x^2}; \qquad \text{e)}\quad \frac{x}{x^2-\alpha^2}; \qquad \text{f)}\quad \frac{x^2+2}{x(2x^2+1)^2}.$$

$$\text{g)}\quad \frac{1}{ax^2+bx+c}; \quad (a,\ b,\ c\in\mathbb{R},\ a>0,\ 4ac-b^2>0).$$

Esercizio 4 Si calcoli una primitiva per ciascuna delle seguenti funzioni, eventualmente utilizzando la sostituzione suggerita:

$$\text{a)}\quad \frac{\sqrt{x+2}}{\sqrt[3]{x+2}+1} \quad (t^6=x+2); \qquad \text{b)}\quad \frac{x}{\sqrt{3x+5}}; \qquad \text{c)}\quad \frac{\operatorname{tg}^2 x+3}{1+\cos^2 x} \quad (t=\operatorname{tg} x);$$

$$\text{d)}\quad \frac{1}{\operatorname{sen} x} \quad (t=\operatorname{tg}(\frac{x}{2})) \qquad \text{e)}\quad \frac{x-\sqrt{\operatorname{arctg} 2x}}{1+4x^2} \quad (t=\operatorname{arctg}(2x)); \quad .$$

Soluzioni. (Da controllare!) 1. a) $\frac{\pi}{6}-\frac{2}{9}$. b) $\frac{2\log(2)-1}{(\log(2))^2}$. c) $4\sqrt{2}-6-4\log\big(3(\sqrt{2}-1)\big)$. d) $\frac{4}{3}$. e) $e-1+4\log(\frac{2}{e+1})$. f) $2\sqrt{\operatorname{senh}(1)+2}-2\sqrt{2}$.

2. a) $\frac{1}{\alpha}\cosh(\alpha x)+\frac{1}{\beta}\sinh(\beta x)$ se $\alpha\neq 0$ e $\beta\neq 0$. b) $(x-1)\log(1-x)-x$.
c) $\frac{1}{2}(x-\operatorname{sen} x\cos x)$. d) $2\big(\operatorname{arcsen}(x/2)+x/2\sqrt{1-\frac{x^2}{4}}\big)$. e) $\frac{1}{2}(\log x)^2$. f) $\frac{1}{5}e^{2x^2}\operatorname{sen}(x^2)-\frac{1}{10}e^{2x^2}\cos(x^2)$.
g) $2e^{\sqrt{x}}(x^{3/2}-3x+6\sqrt{x}-6)$. h) $x\cdot\operatorname{arcsen}(x)+\sqrt{1-x^2}$. i) $x\cdot\operatorname{arctg}(x)-\frac{1}{2}\log(1+x^2)$.

3. a) $\frac{1}{3}x+\frac{1}{9}\log(3x-1)$. b) $\frac{1}{2}\log(x^2+1)+3\operatorname{arctg}(x)$. c) $\frac{1}{64}\log(\frac{x}{x-4})+\frac{1}{16}\frac{x+2}{x^2}$. d) $\frac{1}{3}(x^3-\frac{1}{3x-1})$.
e) $\frac{1}{2}\log(x^2-\alpha^2)$. f) $2\log(x)-\log(2x^2+1)+\frac{3}{4}\frac{1}{2x^2+1}$.
g) La funzione si può scrivere come $\frac{4a}{4ac-b^2}\frac{1}{\big(\frac{2a}{\sqrt{4ac-b^2}}(x+\frac{b}{2a})\big)^2+1}$. Dunque una primitiva sarà

$$\frac{2}{\sqrt{4ac-b^2}}\cdot\operatorname{arctg}\Big(\frac{2a}{\sqrt{4ac-b^2}}(x+\frac{b}{2a})\Big).$$

4. a) $6\big(\frac{(x+2)^{\frac{7}{6}}}{7}-\frac{(x+2)^{\frac{5}{6}}}{5}+\frac{(x+2)^{\frac{1}{2}}}{3}-(x+2)^{\frac{1}{6}}+\operatorname{arctg}((x+2)^{\frac{1}{6}})\big)$. b) $\frac{2}{27}\sqrt{3x+5}^3-\frac{10}{9}\sqrt{3x+5}$.
c) $\operatorname{tg} x+\frac{1}{\sqrt{2}}\operatorname{arctg}(\frac{1}{\sqrt{2}}\operatorname{tg} x)$. d) $\log|\operatorname{tg}\frac{x}{2}|$. e) $\frac{1}{8}\log(1+4x^2)-\frac{3}{4}(\operatorname{arctg}(2x))^{\frac{2}{3}}$.