Regularity of Invariant Sets in Semilinear Damped Wave Equations

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Under fairly general assumptions, we prove that every compact invariant subset \( \mathcal{I} \) of the semiflow generated by the semilinear damped wave equation

\[
eu_{tt} + u_t + \beta(x)u - \sum_{ij}(a_{ij}(x)u_{x_j})_{x_i} = f(x, u),
\]

\((t, x) \in [0, +\infty] \times \Omega, u = 0, (t, x) \in [0, +\infty] \times \partial \Omega\) in \( H_0^1(\Omega) \times L^2(\Omega)\) is in fact bounded in \( D(A) \times H_0^1(\Omega)\). Here \( \Omega \) is an arbitrary, possibly unbounded, domain in \( \mathbb{R}^3 \), \( A u = \beta(x)u - \sum_{ij}(a_{ij}(x)u_{x_j})_{x_i} \) is a positive selfadjoint elliptic operator and \( f(x, u) \) is a nonlinearity of critical growth. The nonlinearity \( f(x, u) \) needs not to satisfy any dissipativeness assumption and the invariant subset \( \mathcal{I} \) needs not to be an attractor.