Università degli studi di Trieste
Corso di Studi in Matematica

Algebra 2 (9 cfu)

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academic year: 2012-2013

1 Preliminary notions

Short summary of the notions of groups, rings, fields, homomorphisms kernel, images, subgroups, ideals, etc. Fermat little theorem, Chinese remainder theorem, Euclid algorithm, Bezout identity (for integers).
Principal ideals, finitely generated ideals, characteristic of a ring, etc.

2 The ring of polynomials in one variable

Construction of the ring of polynomials, division of polynomials, (ir)reducible polynomials, Ruffini theorem, D'Alambert theorem.

3 Factorization of polynomials, part I

Irreducible polynomials in $K[x]$ (K field). $K[x]$ is a UFD. $\mathbb{C}$ is algebraically closed. Irreducible polynomials in $\mathbb{R}[x]$. Derivative of a polynomial and its applications. Perfect fields, Frobenius isomorphism.

4 Factorization of polynomials, part II

Gauss lemma. Theorem of Eisentstein on the irreducible polynomials of $\mathbb{Z}[x]$. CRT for polynomials. Berlekamp algorithm.

5 Polynomials in several variables

Construction and several definitions (degree in one variable, degree in several variables, homogeneous polynomial, etc.). $K[x_1, \ldots, x_n]$ (K field) as a vector space over K. Examples of prime and maximal ideals in $K[x_1, \ldots, x_n]$.

6 Fields

Extension of fields. Transcendental and algebraic elements. Minimal polynomial. Degree of an extension, Tower law. The field $\mathbb{C}$ as the quotient $\mathbb{R}[x]/(x^2+1)$. Splitting field of a polynomial. Construction with ruler and compass.

7 Finite fields

Construction and characterization of finite fields. Moebius function. Number of irreducible polynomials in $\mathbb{Z}_p[x]$.

8 Symbolic computations

An introduction to Sage.

References

- Lindsay N. Childs, *A concrete introduction to higer algebra*, Springer, I edition (1990) and III edition (2009);
- Ian Stewart, *Galois theory*, Chapman & Hall/Crc Mathematics (2004);
- Nathan Jacobson, *Basic Algebra*, San Francisco: W.H. Freeman (1974);
- Israel N. Herstein, *Algebra*, Roma, Editori Riuniti (1992).
- William Stein, *Elementary number theory: Primes, Congruences, and Secrets*, Springer, 2008 - legal free download from: `http://wstein.org/ent/ent.pdf`