Fattorizzazione del polinomio $x^{p^n} - x$ in $\mathbb{Z}_p[x]$
Esempi

- In $\mathbb{Z}_2[x]$:

$$
\begin{aligned}
x^{2^1} - x &= x(x+1) \\
x^{2^2} - x &= x(x+1)(x^2+x+1) \\
x^{2^3} - x &= x(x+1)(x^3+x+1)(x^3+x^2+1) \\
x^{2^4} - x &= x(x+1)(x^2+x+1)(x^4+x+1)(x^4+x^3+1)(x^4+x^3+x^2+x+1)
\end{aligned}
$$

- In $\mathbb{Z}_3[x]$:

$$
\begin{aligned}
x^{3^1} - x &= x(x+1)(x+2) \\
x^{3^2} - x &= x(x+1)(x+2)(x^2+1)(x^2+2x+2)(x^2+x+2) \\
x^{3^3} - x &= x(x+1)(x+2)(x^3+2x+1)(x^3+x^2+2)(x^3+2x+2)(x^3+2x^2+1) \\
&\quad (x^3+x^2+x+2)(x^3+2x^2+x+1)(x^3+x^2+2x+1)(x^3+2x^2+2x+2)
\end{aligned}
$$

- In $\mathbb{Z}_5[x]$:

$$
\begin{aligned}
x^{5^1} - x &= x(x+1)(x+2)(x+3)(x+4) \\
x^{5^2} - x &= x(x+1)(x+2)(x+3)(x+4)(x^2+2)(x^2+3)(x^2+x+1)(x^2+x+2) \\
&\quad (x^2+2x+3)(x^2+2x+4)(x^2+3x+3)(x^2+3x+4)(x^2+4x+1)(x^2+4x+2)
\end{aligned}
$$

Numero di polinomi irriducibili di grado d in $\mathbb{Z}_p[x]$

Con $N(p,d)$ si indica il numero di polinomi monici irriducibili di grado d in $\mathbb{Z}_p[x]$. Ecco alcuni esempi:

d	$N(2,d)$	$N(3,d)$	$N(5,d)$	$N(7,d)$
1	2	3	5	7
2	1	3	10	21
3	2	8	40	112
4	3	18	150	588
5	6	48	624	3360
6	9	116	2580	19544
7	18	312	11160	117648

Infine il numero di polinomi monici irriducibili di grado 200 in $\mathbb{Z}_{101}[x]$ vale:

$$
\begin{aligned}
N(101,200) = \ & 3658008925914970226942294169480417023326465239950059934551199 8\backslash \\
& 9066225367669253087731729702696823580742598837189754641352306 9\backslash \\
& 3980542356283677820423461323749600552333826061564431150037996 8\backslash \\
& 8418623480821430189284649181691846736384237356225696222626522 9\backslash \\
& 8537189995834614643397910624227968304357508494453862787724942 1\backslash \\
& 8254481819318669213148251030282081164654699039166364714218015 7\backslash \\
& 77420595784126774510551804 0
\end{aligned}
$$