Let K be a field (for instance, $K = \mathbb{Q}$ or $K = \mathbb{R}$ or $K = \mathbb{C}$). Let $V = K^4$ and $t : V \longrightarrow V$ given by $t(a, b, c, d) = (-a, d, -c, b)$. Then (V, t) gives a $K[x]$-module M_V. Consider the element $m = (1, 2, -1, 3) \in M_V$. We want to find its order. Then consider

$$m,\ t(m),\ t^2(m),\ t^3(m),\ t^4(m)$$

i.e.:

$$(1, 2, -1, 3),\ (-1, 3, 1, 2),\ (1, 2, -1, 3),\ (-1, 3, 1, 2),\ (1, 2, -1, 3)$$

We see that $t^2(m) - m = 0$, hence the order of m is $x^2 - 1$. In a similar way we can determine the order of any element of M_V, in particular the elements of a system of generators of M_V, which is for instance $(1, 0, 0, 0)$, $(0, 1, 0, 0)$, $(0, 0, 1, 0)$ (the element $(0, 0, 0, 1)$ is not necessary, since it is $x \cdot (0, 1, 0, 0)$). They orders are, respectively: $x + 1$, $x^2 - 1$, $x + 1$. Hence $\mathrm{Ann}(M_V) = (x^2 - 1)$. An element of M_V of order $x^2 - 1$ is $c_1 = (0, 1, 0, 0)$, hence we can decompose M_V as $M_V = \langle c_1 \rangle \oplus L$, for a suitable submodule L of M_V. A basis of $C_1 = \langle c_1 \rangle$ as a K-vector space is $(0, 1, 0, 0), (0, 0, 0, 1)$, hence a basis for L as a K-vector space is $(1, 0, 0, 0), (0, 0, 1, 0)$. Take the element $c_2 = (1, 0, 0, 0) \in M_V$ (and in L). Its order is $x + 1$, hence the cyclic module it generates is $C_2 = \langle c_2 \rangle = \{(u, 0, 0, 0) \mid u \in K\}$. Similarly, the element $c_3 = (0, 0, 1, 0)$ generates a cyclic module $C_3 = \{(0, 0, u, 0) \mid u \in K\}$. Hence we have the cyclic decomposition of M_V given by:

$$M_V = C_1 \oplus C_2 \oplus C_3$$

where C_1 is cyclic of order $g_1 = x^2 - 1$, C_2 is cyclic of order $g_2 = x + 1$ and C_3 is cyclic of order $g_3 = x + 1$. Note that $g_3 | g_2$ and $g_2 | g_1$. The polynomials g_1, g_2, g_3 are the invariant factors of M_V. The endomorphism t, with respect to the basis $t^0(c_1),\ t^1(c_1),\ t^0(c_2),\ t^0(c_3)$ has the associated matrix $U_{g_1} \oplus U_{g_2} \oplus U_{g_3}$, i.e.:

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$