LESSON DIARY
academic year 2025/26

1. Lesson 2025/9/22. Definition of a (commutative, unitary) ring, definition of an ideal of a ring, quotient of a ring, theorems of homomorphisms, zero divisors and unitary elements in a ring. Modules over rings.

2. Lesson 2025/9/25. Module product of modules. Universal property of the product of modules. External direct sum of modules. Universal property of the external direct sum. Free modules. Basis of free modules. A module is free if and only if has a basis. Universal property of the basis of a free module. Exact sequences. Short exact sequences.

3. Lesson 2025/9/29. Module homomorphisms, theorems of homomorphism. Finitely generated modules. Every module is a quotient of a free module. Artinian and noetherian modules, first properties.

4. Lesson 2025/10/2. A module is noetherian if and only if every submodules of it are finitely generated. A ring is noetherian if and only if every ideal is finitely generated. Finite direct sum of artinian (noetherian) modules are artinian (noetherian). Examples of rings that are not noetherian. Hilbert basis theorem.

5. Lesson 2025/10/6. Some consequences of the Hilbert Basis theorem. If K is a field, the polynomial ring $K[x_1, \dots, x_n]$ is noetherian. Greatest common divisor in a domain. Principal ideal domains (PID). In a PID the greatest common divisors between two elements always exists. In a PID an element (not zero and not unitary) is irreducible if and only if is prime. Unique factorization domains (UFD). A PID is a UFD. If R is a UFD, the polynomial ring $R[x]$ is a UFD.
   Cyclic modules over a ring, cyclic modules over a PID. The order of a cyclic module. The annihilator of a module, the minimal annihilator of a module over a PID.

6. Lesson 2025/10/9 A useful short exact sequence. Some lemmas to give the result: If M is a torsion, finitely generated module over a PID and C is a cyclic submodule of M then there exists a submodule L of M such that $M = C \oplus L$. From this it follows the main theorem: Let M be a fintitely generated torsion module over a PID then there exist finite cyclic modules $C_1, \dots, C_k$ of order $\mu_1, \dots, \mu_k$ such that $\mu_2 | \mu_1, \dots, \mu_k | u_{k-1}$ and such that $M = C_1 \oplus \cdots \oplus M_k$.