DISCOVERING OUR HISTORY A HISTORICAL INVESTIGATION INTO MATHEMATICS EDUCATION

Luciana Zuccheri
Department of Mathematics and Computer Science,
University of Trieste, Via Valerio 12/1, 34127 Trieste, Italy
E-mail: zuccheri@units.it
and
Verena Zudini
Department of Psychology,
University of Milano-Bicocca, Piazza dell'Ateneo Nuovo 1, 20126 Milano, Italy
E-mail: verena.zudini1@ unimib.it¹

Keywords: Mathematics Education, History of Mathematics Education, Secondary School

Abstract

In this paper we describe the development of an investigation into the field of the history of mathematics education, based on library material and archival documents. We describe the origins of this study, the phases of the work carried out, the methodologies adopted and the results obtained until now. Such kind of studies are important, because reflecting on the didactical choices made in the past can be useful for improving mathematics education and teacher training itself. The investigation focuses on a particular moment of the history of mathematics education in secondary schools of our place of origin, i.e. the transition from the regulations of the Habsburg Empire to those of the Kingdom of Italy, after the First Word War, in the city of Trieste (now in Italy) and in its surrounding region. The interest of this study is not merely local, in so far as it treats the comparison between the different didactical methodologies used in the two countries (the one more oriented to practical application, the other more theoretical) in the context of the contemporary international situation.

¹ The order of authorship is alphabetical.

Historia vero testis temporum, lux veritatis, vita memoriae, magistra vitae... (Cicero, De Oratore, II, 9, 36)

Introduction

We think that the history of mathematics education, if it is well integrated in disciplinary education, may give remarkable results with respect to learning. With regard to this, one of the authors of the present paper has direct experience and coordinated in the past years an experimentation on the teaching of history of mathematics in secondary schools with high level students by using also direct sources (cfr. Zuccheri, ed., 2008). However, always basing ourselves on direct experience, we consider it most important to study, research and analyse, not only the history of mathematics, but also the history of its education. By that, we mean not only the study of the modification of the teaching programmes of mathematics at different scholastic levels, but also, wherever possible, of the development of the teaching methods of this subject.

Such a cultural operation may deserve attention in itself, but it takes on particular importance in our field of research for its effect on improving mathematics education today. First of all, as would seem obvious, reflecting on the consequences of choices made in the past is useful for preventing mistakes previously made. On the other hand, a correct contextualization of the choices made may also help people understand that some didactical approaches, today considered old-fashioned and mistaken, in other times had a sense and usefulness and as such may be reconsidered more favourably. These kinds of studies may be profitably applied in the field of training of mathematics teachers of any order and level, because promoting a comparison between the methods adopted in the past and today's methods from a historical point of view fosters the development of a critical approach.

In the following contribution we present our investigations in this field and we explain the origins of our interest for such studies, the phases of the work carried out, the methodologies adopted and the results obtained until now.

Origins of this investigation and preliminary phase: the reordering of an archive

The opportunity for comparative studies on different didactical methodologies can occur mostly in territories in which, during relatively recent historical periods of time, sudden or rapid changes have taken place. This occurs, in particular, in the territories where radical political-administrative changes, due to social upheaval or wars, have been effected. An example is the city of Trieste (now in Italy) and its territory, which, at the end of the First World War, moved from being under the administration of the Habsburg Empire (in particular, Austria) to that of the Kingdom of Italy. In order to carry out original and extensive studies, it is also necessary that documentation is available *in loco*, in public archives, public and private libraries, school institutes, universities, etc.

Due to the lucky coincidence of both these conditions and above all to the discovery of unpublished documental material at the Library of the Department of Mathematics and Computer Science of the University of Trieste, we were able to begin an investigation into a subject which was of special interest to us, that is a particular moment of the history of mathematics education in our place of origin. The unpublished material consisted of a collection of documents of the Trieste Section of the Italian Society of Mathematics (from 1922, Italian Society of Physics and Mathematics) "Mathesis".²

We began our investigation carrying out a study (Zuccheri & Zudini 2007a) on this material, which we called "Fondo Mathesis". The archive, of modest sizes, collects documents of the period from 1919 to 1951 and allows the reconstruction, on a general if

3

² On the history of "Mathesis" see its official website: http://www.mathesisnazionale.it and the journal Periodico di matematica per l'insegnamento secondario, thereafter Periodico di matematiche: storia didattica - filosofia, journal of the Society from 1899.

discontinuous line, of the life of the Trieste Section of "Mathesis", from the first Congress of Section's constitution of June 15, 1919, to the beginning of the Fifties of the twentieth century.

The reordering of an archive requires competences in very different fields, which can also be built up gradually, above all by collaboration: specific disciplinary competences are necessary, as well as historical, archival and, possibly, technological competences, for data processing using appropriate software. In our case, we joined our respective competences, besides those in mathematics, to include mathematics education and archival science, in order to carry out a historical investigation.

It should be noted that the documentation available to the scholar's first analysis is often in disarray with no logical order. The scholar must begin with a first phase of recognition and exploration of the material, before getting down to a thorough analysis of it.

The scholar's work will have effect also on the exploitation of the material by the community (scientific and not). Therefore the scholar must work with the agencies and administrations appointed by law to preserve cultural heritage (archives, libraries, museums, etc.), in order to evaluate and manage the documental patrimony, during phases of reordering, inventory and cataloguing, according to the normative in force (such as that of privacy, copyright, etc.). We contacted the archival Superintendence of the Friuli-Venezia Giulia Region and consulted archival experts.

Archival science is the discipline that studies archives in all their forms and problems: how an archive develops in the present time; how past archives were created by different agents (public and private); which methods must be used for the management of a still-developing archive; how an archive is stratified; how the material must be selected; how to work in an already formed archive; how to preserve an archive physically and intellectually; how to carry out access to an archive; what kind of devices and strategies must be used in order to preserve an archive and make it available.

The "field of action" of archival science has recently increased also due to technological innovation, which gives new vital energy to the discipline, from the theoretical point of view too. Who ever deals with archives must be not only a curator of a historical memory, but also an architect of a still-evolving memory, which is being built up day by day.

People generally agree that, despite the varying shades of the different definitions, which change also in the context of the countries in which they are applied, archival science may be defined as a research discipline and as heuristics of documental sources which it studies in their real and historical origin, stratification, transmission, and preservation.

In order to be a faithful memory of its producer, an archive must be not only stored in a place (passive preservation), but its documents must be reworked, rethought, used to analyse the past and plan the future, as the very image of Janus suggests, depicted with two heads looking in opposite directions and adopted as symbol of the archival activity by the International Council on Archives.³

Preserving an archive means not preparing a suitable place for its documents, keeping and restoring their supports, or taking care of its materials, but more importantly creating all the conditions for its administrative as well as scientific exploitation. One of these conditions, perhaps the most important, is the creation of special and integrated instruments (such as the inventory) which allow a link between the documents and the users, a connection between preservation and access. In this sense, the person who works with archives becomes, during the phase of preservation, a sort of cultural mediator between documents and users.

What has been said above about archives is valid also for our "Fondo Mathesis", despite its very simple structure and its small dimensions. We have carried out its reordering and its description, according to the international standards of ISAD(G) (General International

_

³ Official website: http://www.ica.org.

Standard Archival Description) and ISAAR(CPF) (International Standard Archival Authority Record for Corporate Bodies, Persons and Families).⁴

The material which we examined consists of handwritten and typed documents in Italian, and a small part in German, of different kinds (generally, loose sheets, but also copybooks and typed scripts). We carried out the reordering and description maintaining the order in which the documents were found, which does not always correspond to the chronological order. We stored the material in envelopes, in which the documents are sometimes subdivided into files and sub-files.

In order to make the material available to the users, we divided the archive into 4 series, according to a "virtual setting": I. Verbali, materiale assembleare, relazioni e conferenze (Minutes of meetings and reports); II. Carteggio (Correspondence); III. Contabilità e diversi (Statements of money paid and received and other documents); IV. Testi a stampa (Printed documents) (see Figure 1).

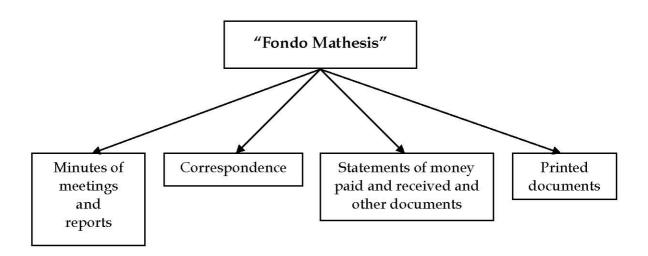


Figure 1. The "Fondo Mathesis"

In the inventory, an item (a sort of small archival sheet) corresponds to each document of the archive. The inventory is in Italian. We give here an English translation of its initial part:

⁴ ISAD(G): http://www.ica.org/sites/default/files/isad g 2e.pdf. ISAAR(CPF): http://www.ica.org/sites/default/files/ISAAR2EN.pdf.

Fonds

Archival marking: Fondo Mathesis

Title: Fondo Mathesis

Name of creator: Trieste Section of "Mathesis" Society

Date: 1919-1951 (discontinuous) Extent and medium: 6 envelopes

I. SERIES

"MINUTES OF MEETINGS AND REPORTS"

Contains:

Marking: Fondo Mathesis, 1

Title: Reports of General Congresses and Board Meetings

Date: 15.6.1919-12.6.1920 **Level of description:** Envelope Extent and medium: 1 copybook

Material characteristics: Handwritten document

Notes: At the end of the copybook there are notes of later date

The inventory goes on with other items, which describe the documents in detail, as shown above, also indicating, in the "Notes", the presence of any particularities.

The development of the investigation

The archive, so rearranged, was a valuable source for beginning an investigation into the change in school institutions, structures and mathematics teaching programmes in the city of Trieste and in the surrounding region (which here and in the following we will indicate with the name "Venezia Giulia",5) after 1918, with the transition from Austria to Italy. For more details on the development of the rule changes we consulted, besides publications on this subject, the documents about the Italian language school institutions in Venezia Giulia, kept at the Archivio di Stato di Trieste (Trieste State Archive), at the Comune di Trieste (Trieste City Council), at the Biblioteca Generale dell'Università di Trieste (General Library of the

⁵ For the geographical and political meaning of the name "Venezia Giulia" see Zuccheri & Zudini 2007b and its references.

University of Trieste) and at the libraries of some secondary schools of Trieste (see Zuccheri & Zudini 2007a).

In Venezia Giulia, there were secondary schools under Austrian administration with teaching in German and also in Italian. These schools all followed the same teaching programmes and the same regulations. With the transition to the Kingdom of Italy the German speaking schools were abolished and the problem arose of adapting the regulations and the teaching programmes to those in force in the Kingdom of Italy. From the reports of the meetings of the "Mathesis" Society, also published in the Bollettino della "Mathesis" (see Cantoni et al. 1920; Furlani 1920b), and mostly from their minutes collected in the "Fondo Mathesis", some elements emerged, which testify to the great activity by the mathematics teachers of the Trieste Italian speaking secondary schools regarding this adaptation. They did it trying to keep the special characteristics of the mathematics programmes in force till then under the Habsburg administration, which the teachers of the Trieste Section of "Mathesis" considered particularly valuable. As it emerges from their reports presented at the National Congress of "Mathesis", which took place in Trieste in 1919 (see Cantoni 1920 and Furlani 1920a), they appreciated above all the Austrian didactic methodology used for teaching mathematics and scientific subjects, which was considerably different from that in force in the schools of the Kingdom of Italy.

We considered this attitude of the Italian mathematics teachers of the secondary schools of Trieste and of its territory remarkable and decided to investigate into its reasons, which, in our opinion, could not be due to their political ideologies at all. On the contrary these ideologies would have pushed them to a completely opposite attitude. In fact, we were aware that many of the protagonists of this event had stood out during the period before the First World War and during the First World War itself on account of their pro-Italian feelings, for which they had paid with sanctions of different severity (which varied from expulsion from teaching to

imprisonment). Regarding that, for some of them, we found evidence also through consulting documental sources⁶ and interviewing their descendants.

We thought it would be interesting to understand the reasons of such a stance taken by the Italian teachers. So we decided to intensify our investigation and we compared the local with the contemporary international situation (see Zuccheri e Zudini 2007b).

The beginning of the twentieth century was a fundamental moment for the development of mathematics education. It was a fertile period in research and thorough examination of teaching problems, in particular in the Middle-European cultural world. The Fourth International Congress of Mathematicians held in Rome (on April 6-11, 1908) was the occasion for analysing and discussing the status of mathematics teaching in various countries, with reference also to psycho-pedagogical considerations. On that occasion, an International Commission for the study of teaching problems was set up. It was called "Commission Internationale de l'Enseignement Mathématique" (CIEM) or "Internationale Mathematische Unterrichtskommission" (IMUK) (thereafter, "International Commission on Mathematical Instruction" (ICMI)⁷), whose centenary was celebrated during a Symposium held again in Rome⁸. Its target was to promote and spread the interest of the mathematicians in school education. Its first President was the German mathematician Felix Klein (1849-1925), who promoted at that time an important reform in mathematics teaching in Germany. The Commission, together with the journal L'Enseignement Mathématique, played a very important part as trait d'union among scholars, allowing their communication and cooperation at an international level (see Schubring 2003; Furinghetti 2003; Schubring 2008; Furinghetti et al. 2008).

In Austria, a reform in school curricula performed by Minister Gustav Marchet, came into

⁶ See Schiffrer & Furlani 1964-1965; Comune di Trieste, Servizio Bibliotecario Urbano, Archivio Diplomatico, "Scuole del Comune – Periodo bellico – Personale politicamente infido all'Austria (cioè di sentimenti italiani)", 13 D 38/2; *Annuario del Ginnasio Superiore Comunale (Ginnasio-Liceo)*

[&]quot;Francesco Petrarca" di Trieste 1918-1919, pp.6ff.

⁷ For more details: http://www.icmihistory.unito.it/officers.pdf.

⁸ Cfr. Menghini et al. (eds.) 2008. Official website: http://www.unige.ch/math/EnsMath/Rome2008.

effect in the school year of 1909/1910. That reform arose from the need to take into account the progress of science but also to modify the conception of teaching and its methods. It was inspired by principles such as the adjustment to the pupils' grade of intellectual development, the simplification of courses through a closer contact among the different branches, especially between arithmetic and geometry, the complete adjustment of the mathematical study to corresponding disciplines and to everyday life. These principles were related to Felix Klein's programme in order to improve the teaching of mathematics.

At that time in the Kingdom of Italy, where, during the process of unification, also in reaction to long years of living under foreign dominion, mathematicians had decided to plan the mathematics teaching programmes, rejecting the methodologies proposed in the foreign textbooks used before and according to strict classicist criteria, the time was ripe for school reform too; it was viewed as necessary also in response to the country's industrial and technical progress. In Italy a reformist current, which advocated a less theoretical and a more modern way of teaching, made itself felt. However, at the end of the First World War, no global reform in that sense had been effected yet (see Giacardi 2006).

The problem of changing the teaching programmes in the former Austrian territories, among which Trieste and its territory, annexed to the Kingdom of Italy at the end of the First World War, was not of local interest, but was in response to the demands of global renewal which affected all contemporary mathematics education in Italy, in the context of the European movement mentioned above. The National Congress of the "Mathesis" held in Trieste on October 1919 gave the opportunity for a comparison between the programmes and the methods used till then in mathematics teaching in the Kingdom of Italy and in the newly annexed territories. Various reports were held on the subject, concerning the different types of schools, with attention to cognitive and learning aspects.

On that occasion the different choice of topics and the different distribution of contents in the various school years and levels were highlighted. In some cases stark differences emerged. For example, as an effect of the afore-mentioned reform by Marchet, in the Austrian secondary schools the first elements of differential and integral calculus had been already introduced; these were not included, with rare exceptions, in the official teaching programmes of the schools of the Kingdom of Italy. The importance of these subjects in secondary teaching, already pointed out in the EM-ICMI Symposium held in Geneva in 2000 (see Coray et al., eds., 2003; and in particular, Kahane 2003), directed our investigation also into the teaching of calculus in the city of Trieste before the First World War (see Zuccheri & Zudini 2007c).

In the reports presented by the teachers of the Trieste Section of "Mathesis" at the Congress of 1919 it was evident, moreover, that the principal difference between Austria and Italy in teaching mathematics (and scientific disciplines) lay, in general, in the methodology used, due to deep-set school principles. In Austria a great emphasis was put on exercises, with the purpose of learning based on practice and on practical applications of theoretical subjectmatter, while in Italy more attention was given to a more theoretical and formally strict teaching, in geometry as well as in arithmetic. Thereafter, this difference was further delineated by some secondary school teachers of the Trieste Section of "Mathesis", who were sent to Rome and to Bologna to study the mathematics teaching methods adopted in the senior secondary schools of those cities in the Kingdom of Italy. They presented a report in which they underlined that both didactic tendencies, if taken to the extreme, could have negative effects: on the one hand, a too theoretical teaching, such as that in Italy, could lead a still immature student to repeat merely what he had learned, without actually comprehending its meaning; on the other hand, excessive attention to the exercises, at the expense of the theories and the rigour of proof, as happened in Austria, could limit a student to the study of applications of formulae and to pure calculus, and that could degenerate into a useless mechanical approach. The teachers concluded that the ideal solution would be a gradual closing of the gap between the two teaching systems adopted till then under the two

administrations, through the pursuit of synergy by the Italian teachers of the whole Kingdom of Italy, in order to achieve a "right" interpenetration, keeping the good points from both sets of methods (see Cantoni et al. 1920).

On the other hand, the "Mathesis" Congress of 1919 had concluded with the vote that the Ministry of Public Instruction should not immediately change the mathematics teaching programmes in Venezia Giulia, but take into account the programmes and methods used there till then and should study their advantages, with a view to a much-needed general revision of the programmes of the secondary schools in the whole Kingdom of Italy.

Encouraged to propose their ideas for the gradual transition to the programmes of the Kingdom of Italy, in view of a reform that was supposed to take into consideration their experience, but that did not ever occur as it had been wished⁹, the Trieste "Mathesis" teachers acted on the basis of their didactic and pedagogical convictions, which were inspired by the principles held by Felix Klein (for more details on these principles see, for example, Klein 1925), and which they had tested during actual teaching.

Our investigation focused thereafter on identifying theoretical principles on which the methodologies used in secondary mathematics teaching in Trieste before the First World War were based. We had already discovered during previous research (in Voghera 1922; in documents of the "Fondo Mathesis"; and in a report of the teacher meetings of a secondary school, kept at the State Archive of Trieste) the appreciation of the Trieste "Mathesis" teachers for a method which they called "Jacob Method" after the name of its author. Our discovery in a library of a secondary school in Trieste of the original text in which this method is described by Jacob himself (Jacob 1913) gave us the opportunity to begin our study (see Zuccheri & Zudini 2007c and 2008).

12

-

⁹ In fact, in 1923 the Minister of the first fascist government Giovanni Gentile carried out a blanket reform of the whole Italian school system, based upon principles which did not take into account the contemporary reformist trends in the teaching of scientific disciplines at all.

Josef Jacob, a school director in Vienna, was very interested in the problems of mathematics teaching in the secondary school and also wrote textbooks which were translated into Italian by a teacher of the Trieste Section of "Mathesis". The book by Jacob, which we studied, illustrates a practical mathematics teaching method, which covered all the subjects treated in the Austrian "Gymnasium" (students aged 11–18). The preface to this book was written by the famous Austrian physicist and philosopher Ernst Mach (1838-1916), by whose cognitive theories, and first of all by the *general principle of economic function of science* (seen as a sort of "thought economy"), Jacob was inspired during the writing of the book itself. In his book he made references mostly to Mach 1889 and Mach 1906; there are other references also to pedagogical works by other authors (Treutlein 1911 and Höfler 1910).

The method, which followed principles in tune with those proposed by Klein and by the Commission CIEM/IMUK, aimed to achieve a contemporaneous and harmonious development of the three students' faculties (i.e. intuitive, deductive and creative), corresponding to the abilities of their age. It focused on the student rather than on the subject, concerning itself with effective comprehension rather than with mathematical rigor.

Jacob's treatise went on giving very precise didactical examples, explaining to the teachers, plainly and in a practical way, how to introduce any subject. This was carried out from the viewpoint of secondary mathematics teaching with the aim of making the mathematical "facts" and their interrelations comprehensible to young people as briefly as possible, with simple and schematic representations, of training in conceptual thought, by encouraging intuition and avoiding premature abstraction, as well as of increasing the value of mathematics by applying it in practical life, in technology and in science.

Conclusions and possible developments

The study which we have carried out allowed us to begin a thorough analysis of the question of the change in mathematics teaching programmes and methodologies in Trieste

and in the surrounding region in the period of transition, after the First World War, from the Austrian administration to that of Kingdom of Italy. As we noted, such a question was of great interest, due to both the characteristics of Trieste, a "frontier city" (Ara & Magris 1987²), and the particular moment in the history of mathematics teaching in the Italian and European context. Regarding this, it may be said that Venezia Giulia played for a short period of time the role of a didactic "laboratory", in so far as there had been already for a long time in place programmes and educational methodologies more suited to the new innovative ideas already at the fore in the international context, compared to those normally used in the rest of the Kingdom of Italy.

This deeper analysis carried out also from the disciplinary point of view allowed us to highlight some aspects, first of all the methodological ones, which could have been obscured by the mere reading of the teaching programmes adopted concerning mathematics in the region under consideration, before and after the First World War. Limiting to such a reading (as it was done, for example, by scholars, who, all the same, carried out thorough valuable historical investigations into institutional aspects of schools of the region) and observing only the continuity of some subjects, the abolition or the substitution of other ones, or their moving to other stages of the curriculum, without investigating into the reasons, results in being unable to highlight all the questions regarding any deep-set methodological differences (i.e., for example, those which at that time the teachers had to face and which shaped the future development of mathematics education in the region).

Our investigation will go on in this direction, because we think that such studies, although carried out at local level, are useful for rebuilding the history of mathematics education at a more general level, with profitable effects also on teacher training, as we said in the introduction of this paper. Therefore we wish that similar investigations may be carried out in other regions which experienced historical events similar to those examined by us.

Our studies were possible thanks to the existence of documental and library material which fortunately had been preserved. During our investigations we found out that a lot of other material, which could have been important for our purpose, had unfortunately not been preserved. We consider that making available to teachers and future teachers the results of investigations like this may be useful also for fostering the development of a sensitivity to the preservation of documents, which in school administrations is often felt as an imposed, heavy and meaningless duty. In our opinion our investigation has shown that well arranged and managed documentation may be a cognitive and operative resource.

Acknowledgements

We wish to thank Gert Schubring for his useful suggestions and comments on this investigation.

References

- Ara, A., & Magris, C., 1987², Trieste. Un'identità di frontiera, Torino: Einaudi.
- Cantoni, A., 1920, "Programmi e metodi dell'insegnamento nelle scuole delle terre redente e negli antichi confini d'Italia", *Bollettino della "Mathesis"* **12** (1-4), 8-17.
- Cantoni, A., Furlani, G., Nadalini, G., & Verson, A., 1920, "Relazione riguardante l'insegnamento della Matematica", in Relazioni sull'insegnamento della Matematica, della Fisica e della Chimica, *Bollettino della "Mathesis"* 12 (9-12), 165-173.
- Coray, D., Furinghetti, F., Gispert, H., Hodgson, B. R., & Schubring, G. (eds.), 2003, *One Hundred Years of L'Enseignement Mathématique. Moments of Mathematics Education in the Twentieth Century*. Proceedings of the EM–ICMI Symposium (Geneva, 20–22 October 2000), Geneva: L'Enseignement Mathématique.
- Furinghetti, F., 2003, "Mathematical instruction in an international perspective: The contribution of the journal *L'Enseignement Mathématique*", in D. Coray et al. (eds.) 2003, pp. 19–46.
- Furinghetti, F., Menghini, M., Arzarello, F., & Giacardi, L., 2008, "ICMI Renaissance: The emergence of new issues in mathematics education", in M. Menghini et al. (eds.) 2008, pp. 131–147.
- Furlani, G., 1920a, "Rapporti fra la matematica e la fisica nell'insegnamento", *Bollettino della "Mathesis"* **12** (1-4), 22–31.
- Furlani, G., 1920b, "Relazione sull'insegnamento della matematica del prof. Giacomo Furlani alla Sezione romana della "Mathesis,,, 2 maggio 1920", in Verbali delle Sezioni, *Bollettino della "Mathesis"* **12** (9-12), 176–182.
- Giacardi, L., 2006, "L'insegnamento della matematica in Italia dall'Unità all'avvento del fascismo", in *Da Casati a Gentile. Momenti di storia dell'insegnamento secondario della matematica in Italia*, L. Giacardi (ed.), Lugano: Lumières Internationales, pp. 1–63.
- Höfler, A., 1910, Didaktik des mathematischen Unterrichts, Leipzig: Teubner.
- Jacob, J., 1913, *Praktische Methodik des mathematischen Unterrichts*, Wien: A. Pichlers Witwe & Sohn (Preface by E. Mach).

- Kahane, J. P., 2003, "L'enseignement du calcul différentiel et intégral au début du vingtième siècle", in Coray et al. (eds.) 2003, pp. 167–178.
- Klein, F., 1925³, *Elementarmathematik vom höheren Standpunkte aus*, *II. Band. Geometrie*, Berlin: Springer (1st Edition 1908).
- Mach, E., 1889, *Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt.* 2nd Edition, Leipzig: Brockhaus (It. Edition *La meccanica nel suo sviluppo storico-critico*, Torino, Boringhieri, 1968).
- Mach, E., 1906, Erkenntnis und Irrtum. Skizzen zur Psychologie der Forschung. 2nd Edition, Leipzig: Barth (It. Edition Conoscenza ed errore. Abbozzi per una psicologia della ricerca, Torino, Einaudi, 1982).
- Menghini, M., Furinghetti, F., Giacardi, L., & Arzarello, F. (eds.), 2008, The First Century of the International Commission on Mathematical Instruction (1908-2008). Reflecting and Shaping the World of Mathematics Education, Roma: Istituto della Enciclopedia Italiana fondata da Giovanni Treccani.
- Schiffrer, C., & Furlani, G., 1964–1965, "Le lotte per l'Università italiana in Austria", Off-print from *Trieste*, **11** (64) and **12** (65).
- Schubring, G., 2003, "L'Enseignement Mathématique and the first International Commission (IMUK): The emergence of international communication and cooperation", in D. Coray et al. (eds.) 2003, pp. 47–65.
- Schubring, G., 2008, "The origins and early incarnations of ICMI", in M. Menghini et al. (eds.), 2008, pp. 113–130.
- Treutlein, P., 1911, Der geometrische Anschauungsunterricht, Leipzig/Berlin: Teubner.
- Voghera, G., 1922, "Intorno ad un metodo di insegnamento della matematica in uso nelle scuole delle terre redente", *Periodico di matematiche: storia didattica filosofia*, **S. IV**, **Vol. II** (5), 475–478.
- Zuccheri, L., & Zudini, V., 2007a, Animi divisi. Vicende dell'insegnamento della matematica nella Venezia Giulia dal 1918 al 1923, Trieste: EUT.
- Zuccheri, L., & Zudini, V., 2007b, "Identity and culture in didactic choices made by mathematics teachers of the Trieste Section of 'Mathesis' from 1918 to 1923", *The International Journal for the History of Mathematics Education*, **2** (2), 39–65.
- Zuccheri, L., & Zudini, V., 2007c, "On the influence of cognitive theories in the teaching of calculus in Austrian secondary schools at the beginning of the 20th century", *Rendiconti dell'Istituto di Matematica dell'Università di Trieste*, **39**, 347–357.
- Zuccheri, L., & Zudini, V., 2008, "The 'Jacob Method': an example of application of cognitive theories in the period of the introduction of calculus in Austrian secondary mathematics instruction", *The International Journal for the History of Mathematics Education*, **3** (2), 57-64.
- Zuccheri L., ed., 2008, "Metodi della matematica attraverso i tempi", in *Con le mani e con la mente. I laboratori di matematica del Progetto Lauree Scientifiche dell'Università di Trieste*, E. Mezzetti (ed.), Trieste: EUT, pp. 144–184.