Regularity of Invariant Sets in Semilinear Damped Wave Equations

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Under fairly general assumptions, we prove that every compact invariant subset $\mathcal{I}$ of the semiflow generated by the semilinear damped wave equation

$$\epsilon u_{tt} + u_t + \beta(x)u - \sum_{ij}(a_{ij}(x)u_{x_j})_{x_i} = f(x,u),$$

$(t,x) \in [0,+\infty[\times\Omega, u = 0, (t,x) \in [0,+\infty[\times\partial\Omega$ in $H^1_0(\Omega) \times L^2(\Omega)$ is in fact bounded in $D(\mathbf{A}) \times H^1_0(\Omega)$. Here Ω is an arbitrary, possibly unbounded, domain in R^3, $\mathbf{A}u = \beta(x)u - \sum_{ij}(a_{ij}(x)u_{x_j})_{x_i}$ is a positive selfadjoint elliptic operator and $f(x,u)$ is a nonlinearity of critical growth. The nonlinearity $f(x,u)$ needs not to satisfy any dissipativeness assumption and the invariant subset $\mathcal{I}$ needs not to be an an attractor.